Modeling a Channel Migration Corridor for the 59-Mile Segment of the Missouri National Recreation River

Final Report

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January 4, 2009

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1.0 Abstract

This project represents an attempt to predict a 100 year migration corridor for the 59-mile segment of the Missouri National Recreational River from Gavins Point Dam to Kensler's Bend (approximately from Yankton, South Dakota, to Ponca, Nebraska). This prediction is based primarily upon the results of running multiple river simulations using the Johannesson-Parker 1989 meandering model. Several computer programs were written in Python (also using the WxPython and NumPy libraries) to digitize the banks of the Missouri river from USGS topographic maps and Google Earth satellite images, to calculate the river centerline and width, to digitize the river valley wall boundaries, to perform the meandering simulations while maintaining river points within the simulation boundaries, and to check for and remove loop cutoffs (switchbacks in the river which touch or cross one another). These programs were used to run many (3612) slightly different 100 year simulations in order to accumulate 2d valley coverage statistics showing the individual and aggregate locations of the various simulated rivers over time, and to perform a sensitivity analysis of river evolution with respect to input parameter values. This data will be useful in planning river management and land use during the next century, and in assessing the impact of the river on the proposed growth of structural and population development within the valley. In addition, a simple multiple-thread river model is presented as an example of future research on this topic.

2.0 Introduction

SUMMARY -- The initial objective of this study was to predict the lateral motion and migration of the 59-mile segment of the Missouri National Recreational River by simulating the evolution of the geometry of this segment using one or more currently published meandering models. The study would attempt to answer the following question: given the initial configuration of the river at the present day, how far from this configuration would the simulated river travel, and in which directions, during the next 100 years? However, this goal proved to be impossible to attain due to the inability of current 'state of the art' meandering models to adequately represent and predict the motion of the actual river in a single simulation, making use of digitized bank coordinates and measured physical values for input parameters. It was clear that to accomplish this goal would require the developent of new models of river meandering and evolution which were beyond the scope of the present study.

Instead, we sought the simplest possible solution that still captured most of the physics of the river's migration, but making use of a large set of similar but slightly different simulations using the traditional methods to yield an aggregate probabilistic result. Thus, rather than attempt to predict the evolution of a single simulated river with great accuracy, it was decided to make use of a large set of simulations, each having moderate accuracy, to predict the evolution of a population of similar rivers, and to make statements about the likely evolution of a single river based on the behavior of this population. Specifically, the '100 year migration corridor' was to be determined from the fraction of this population of simulated rivers which reached and occupied various points within the river valley during the course of the simulation, and from the boundaries that were defined by the limits of these regions and the area they enclosed.

BACKGROUND -- The Missouri River is the longest river in the United States, estimated at 2,540 miles (4,090 km) for the total length of all reaches. From its origins in Montana and Wyoming until it joins the Mississippi River near St. Louis, the Missouri acquires about 1/6 of the total surface runoff water from rain and snow in the continental US. During the past 200 years, the shape and behavior of the Missouri River has been changed considerably by human intervention aimed toward the development of flood control, navigation, irrigation, hydroelectricity, water supply, and recreation. While the Missouri River of 200 years ago was generally free-flowing and unpredictable, the river of today is tamed by 600 miles of reservoirs and 800 miles of artificial channels, and six dams which limit its flow and sediment content. However, in 1978, 59 miles of the river from Gavins Point Dam, near Yankton, South Dakota, to Ponca, Nebraska, were designated as a nationally protected area where the conditions and environment of the river would be maintained closer to its free-flowing and environmentally-friendly ancestry. In 1991, another 39 miles of the river from Fort Randall Dam to Running Water, South Dakota, were also included in this mandate, the two segments together being known as the Missouri National Recreational River.

The present study is concerned only with the 59-mile segment of the MNRR, which is considered to be the 'wilder' and 'more natural' of the two segments. This segment of the MNRR, on the South Dakota and Nebraska state border, remains closer to the shape, appearance, and behavior of 200 years ago than do other segments of the Missouri River, and it is relatively free to meander across the surrounding valley much as it did before human intervention and deliberate engineering. Although there are currently several flow regimes in this segment which have been altered artificially, part of the intent of the MNRR designation was to preserve the free-flowing condition as much as was practical to do so, given the often conflicting constraints and desires of the local population and various group entities which manage the river. There has been considerable success in this respect, as the 59-mile segment of the MNRR today has numerous islands and small secondary channels, submerged and surface sand bars (with and without persistent vegitation), and naturally eroding banks and surrounding topography similar to the historical river.

Land area adjacent to the MNRR is owned both publicly and privately in Nebraska and South Dakota. Many individuals and groups are engaged in activities intended to curtail or repair bank erosion, maintain ecologocal habitats for indigenous species (including those which are threatened or endangered), and generally improve recreational opportunities on the river. Nevertheless, these activities do not always coexist harmoniously, nor do they have the same goals in mind. For example, while about one third of the banks along the 59-mile segment of the MNRR have been artificially stabilized in some manner since 1978 --mostly to protect privately own land--, during the same time period, naturalists and ecologists have come to realize that channel migration and bank erosion are necessary for creating and maintaining the aquatic and riparian habitats of the river, and that curtailing or halting the meandering of the river would entail the destruction of many of these habitats and preclude the formation of new ones. Similarly opposing viewpoints exist concerning the costs and benefits of water level and flow rate, and potential alterations to bed geometry for purposes of recreation and navigation.

Most land adjacent to the MNRR is privately owned, while the remainder is divided among tribal, state, and federal entities. Managing the natural resources within and near to the river in a way that is amenable to all viewpoints is quite difficult. Currently, ecological habitats within and adjacent to the MNRR contain at least 237 bird species, 80 fish species, 17 reptile species, nine amphibian species, and more than 40 mammal species in addition to Homo Sapiens. Threats to these habitats and other natural resources include fragmentation and loss due to residential, agricultural, and other commercial development, pollution from agriculture, industry, and recreation, the construction and maintenance of artificial erosion-control structures, and the introduction of invasive species from other ecosystems, as well as disputed management issues concerning the control of water level, flow, and river migration appropriate to different uses of the river. In addition, shoreline areas are disappearing at an accererating rate as more primary dwellings, vacation homes, and other structures are built on desirable river-front property.

Before the construction of six major dams changed the natural variation in flow due to seasonal runoff, the Missouri River was prone to periodic flooding, bank erosion, and concomitant sediment transport and redeposition that lead to considerable temporal and spatial variation in the river channel, shallows, and surrounding floodplain. Although the river carries considerably less sediment volume today, it still retains the nickname "Big Muddy", and can be clearly distinguised from the Mississippi River where the two join in St. Louis, due to its higher sediment content. Historically, river migration, erosion, and sediment redistribution was responsible for sandbar formation and destruction which created transient habitats, washed others away, and created new ones elsewhere. This resulted in a constant turnover of rich and diverse populations of plants and animals within and along the banks of the river.

After 1944, as a result of the construction of dams, and the consequent regulation and attenuation of both the averge flow and its high seasonal variation, the Missouri River lost many of its most characteristic features, including braids, islands, sandbars, chutes, and the oxbow lakes formed from previously cutoff segments of the meandering river. These modifications have in turn been responsible for the loss of aquatic and riparian habitats along the river, and declines in ecosystem diversity, stability, and longevity have been the overall result. Since the natural flows, floods, and their associated ecological benefits have been somewhat curtailed, the relatively free-flowing segments of the MNRR have become particularly important, especially in so much as they are permitted to continue to meander uninhibited across the planform of the valley. There has been increased recognition of and interest in the importance of these segments in terms of their ability to resist habitat loss and further damage to the ecology of the river, and in preserving undisturbed ares which still remain.

This report is based on the hope that an improved understanding of the spatial and temporal variability of MNRR geometry and dynamics can contribute important information to those people who make decisions about the river and the management of its resources. Specifically, a quantitative prediction of channel migration that delineates the probable limits --both nominal and extreme-- of river motion during the next century should provide managers with necessary information to assess proposed intervention activities according to the inherent characteristics of the habitats and other resources contained within those regions. In the case of bank-stabilization decisions, for example, this prediction will delineate reaches in which channel migration rates are inherently higher or lower than the norm, and where there is more or less danger to the loss of surface area, sediment, or biomass. Such information can be used to determine where channel migration can be tolerated and where channel migration presents unacceptable risks or conflicts with other uses of the river.

An understanding of the process of river meandering is essential to an understanding of river processes in general. That general understanding is important for people whose work covers a broad range of spatial and temporal domains and their transitions on scales from the erosion of pasture land from year to year to the evolution of an entire river basin over geologic time, with the consequent changes in ecological communities and niches which are a function of these transitions. However, based on observations of the geometry of the MNRR made from maps and satellite photographs, and comparisons of this geometry to that produced by meandering simulations, we have found that important aspects of migrating behavior were not represented in or predicted by the currently published 'state of the art' models, especially the Johannesson-Parker 1989 model used in this study. This is primarily due to the fact that such models represent the river as an idealized single channel having only a centerline and width, and a trapezoidal channel cross-section, rather than their failure to include some essential aspect of the physics in their equations of motion.

Current meandering models do not take into consideration any of the 3-dimensional aspects of the topography of the river bed or banks and surrounding shallow areas, specifically the many islands and submerged and surface bars which are contained within the 59-mile segment of the MNRR. We feel that in order to accurately understand and predict the evolution of a river such as the MNRR, a model which is aware of this 3-dimensional topograpy and the multiple channels, chutes, and braids which contribute to the overall flow and migration of the river is essential. Although the development of a new meandering model of this type is beyond the scope of this project, we present the results of some initial attempts at multiple-thread simulations in the discussion section 5.4.3.1 later in this report which indicate the direction in which future research might go.

CONCLUSION -- The objective of this study was to develop a quantitative prediction of motion of the 59-mile segment of the MNRR that delineates migration domains and ranges which are amenable to potentially different management strategies. The main tool to accomplish this objective was the Johannesson-Parker 1989 meandering model, considered to be the nominal mathematical approach for such studies. Although this tool was not sufficient to develop a prediction based upon a single simulation of the river, we feel that it was sufficient to do so when employed in a novel way --specifically to generate a large population of simulations, each of which contributed a small part to the overall prediction. Nevertheless, the present work leaves some questions unanswered relative to the relationship between the river and its representation as a simple mathematical model. To answer these questions, the development of a multiple-thread meandering model more appropriate to the specific characteristics of the MNRR should be pursued as a subsequent project.

The Missouri National Recreational River preserves two segments of some of the last free-flowing portions of the once-wild Missouri River. The eastern 59-mile reach --with its wide, meandering channel, shifting islands and sandbars, and many small secondary channels-- contains some of the best natural aquatic and riparian landscape associated with the Missouri River along its entire length, and perhaps the last naturally preserved floodplain and wetland habitats on the river or anywhere on the eastern border of the Great Plains. The MNRR today represents a diverse and dynamic ecosystem whose future is uncertain. Both care and well-informed deliberation should be employed in its management and use for several generations to come.

2.1 Study location

The 59-mile segment of the Missouri National Recreational River is situated at (and defines) the eastern border between South Dakota and Nebraska. This reach of the river extends from Gavins Point Dam (at the eastern edge of Lewis and Clark Lake), near Yankton, South Dakota, south-eastward for approximately 95 river kilometers to the city of Ponca, Nebraska. The inlet at the dam is at longitude -97.480463 decimal degrees, latitude 42.850196 decimal degrees, and the 'outlet' at Ponca (an arbitrary point delimiting the south-eastern end of the river in this study) is at longitude -96.647947 decimal degrees, latitude 42.560457 decimal degrees.

<image caption='Map of study area from maps.google.com'>

<image caption='Satellite mosaic made from images at earth.google.com'>

The maps and satellite images used in this study will be discussed in more detail in section 3.2.

After leaving the concrete inlet at Gavins Point Dam, the river is relatively free to meander bewteen the southern wall of the valley (which rises above 1300' near the river and to over 1500' beyond it) and the relatively flat plain to the north, before it enters the concrete channel wall and and wing dikes of Kensler's Bend, and then continues on through Sioux City, Iowa. The inlet is at river mile 811 and an elevation of 354.76 m, while the outlet --almost 100 km away-- is at river mile 753 and an elevation of 336.40 m. The surrounding area is relatively flat to within 20 m. The extent of this valley, and the limit to the potential migration range of the river, was defined in this study by the approximate 1200' (365.76 m) topographic line.

<image caption='Elevation of study area from National Map'>

Here is the elevation of the river and valley in 3d, with the z axis exaggerated by a factor of Y.YYx:

<image caption='Study area in 3d using National Map Digital Elevation Model'>

2.2 Dynamics of river meandering

Meandering rivers and streams are familiar features of the Earth's landscape. The topography of meandering flows often consist of relatively flat valley or canyon floors, as in the case with the 59-mile segment of the MNRR. There are also meandering flows which incise vertically down into the surrounding landscape, creating terraces at different depths, but in this study we are concerned only with meandering motion which occurs in 2-dimensions primarily at a single depth near to the surface level. In these cases, channel migration tends to flatten the valley bottom over time by net erosion of the banks and deposition of alluvial sediment carried by the flow from upstream or other parts of the valley. Thus, the transport of sediment from one point to another, either along the downstream direction of flow, or cross-stream in the direction of channel migration, is an important function of the meandering process.

Although it is perhaps natural to expect that flowing water would prefer to travel in a straight line, this is actually not the case, and it is rare to find a river running in a straight line for very long. From above, rivers usually look like convoluted lines consisting of many loops which frequently change direction and curve back upon one another. The origin of the term "meander" is reputed to be the name of the Menderes river in present-day Turkey, although the ancient Greeks knew this as Maiandros or Maeander. This river is characterised by a very convoluted path along the lower reach, and has a flat flood plain which is much wider than the currently active meandering zone.

A 'meander' in general is a bend in a sinuous watercourse, although 'to meander' means simply to move in a somehat erratic or inconstant maner, changing direction often. A meander is formed when the moving water in a river erodes the outer banks and widens its channel. The material removed from the migrating bank is carried by the flow and made available for potential redeposition somewhere downstream. A river of any size may exhibit a meandering course, alternatively eroding sediment from the outside bank of a bend and depositing it elsewhere (possibly on the inside bank), and then from the opposite bank as the newly formed meander causes a change in the direction of the bend. The result is a snaking pattern of loops as the stream wanders back and forth across its valley. Over time, meandering loops generally tend to migrate downstream with the flow.

<image caption='Example of meandering river'>

When a meandering loop curves back upon itself sufficiently so that the channel intersects itself, the main flow of the river then takes a 'shortcut', excising the loop from the main stream, and an 'oxbow' lake is formed, named for the characteristic crescent shape of the now unmoving water remaining in the isolated basin. As additional bends form and are cut off, many oxbow lankes will be left behind as a legacy of previous locations of the river, and will eventually fill with vegitation and other biomass, or will dry up. However, rather than intersecting itself, a recurving loop may suddenly change direction again and continue to migrate, forming a 'compound bend' consisting of two or more sequential loops linked together.

These two mechanisms --cutting-off and compound bend formation-- are both important for the development of complex meandering stream patterns. Bends removed when the channel bypasses them by seeking a shorter path across their intersection lead to the main axis of the flow shifting to one side or another at different locations, and the channel course becomes erratic as the discarded loops are shed first from one side of the flow, and then from the other. In the absence of cutoffs, bends continue to grow and form compound bends, increasing the overall length of the river until they too are eventually excised. In this way, the length of a river will grow and shrink again in bursts, as new bends and compound bends are formed which gradually increase the overall length, and are then cut off and removed all at once.

<image caption='Evolution of meandering river shape and length'>

The meander-generating process in each river is both dynamic and unique. River meanders develop throughout the years, and continue to change over long periods of time, perhaps never converging on a stable or quiescent pattern. The perpetual creation of meanders is a natural phenomenon which occurs similarly in rivers all around the globe, so that river meandering is one of the most predominant geometric phenomena on the surface of Earth. The physical processes encountered and created by meandering rivers have long intrigued scientists and engineers alike. Not only does the characteristic shape of river bends and beds give rise to a complicated two- or three-dimensional water flow field, but --equally important--, the flow field reshapes the bends and bed surface through the process of erosion, indicating the existence of a complex feedback loop between water flow and bed topography.

An intriguing observation about meandering geometry and dynamics is the similar basic shape of all meanders, despite the different geophysical locations, conditions, and material properties of rivers around the world, and that these similarities can be captured in mathematical models and computer simulations. One common, and possibly counter-intuitive, characteristic shared by meandering rivers all around the world, appears to be the adage that --similar to 'nature abhors a vacuum'-- 'flowing water abhors a straight line'. This observation is often considered surprising (although it should be familiar to anyone who has handled a wildly undulating garden- or fire-hose), and begs the question of why water conduits of any size become sinuous in the first place.

The answer appears to be that water flowing in a straight line is an example of an unstable equilibrium --similar to that of a pencil balanced upon its point-- which does not endure for long. An equilibrium is unstable because any slight perturbation away from that state causes additional perturbations of an increasing magnitude. In the case of a meandering river, what contributes to this instability is the observation that water (or any mass) flowing or moving through a conduit which changes direction (as in around a curve or bend) tends to act in a manner which exaggerates that change in direction, and therefore the shape of the curve over time. There are at least two possible explanations for this phenomenon, which lead to a general division bewteen the 'forced based' and the 'speed based' models of meandering.

This instability, and its positive feedback, can be demonstrated by a simple simulation of meandering in which the rate at which points on a river migrate laterally cross-stream to the direction of the flow is a simple multiple of the signed curvature of the river at that point. Curvature can approximately be considered as the relative amount the river changes direction in angle per unit of its length (where angles to the left are considered 'positive', and angles to the right 'negative'), although in this study curvature is calculated specifically as the inverse of the radius of a circle passing through every three sequential points of the river (see sections 3.4.1.1. and 3.4.1.2 for details). Thus, gentle bends in the river have a small curvature, sharp bends have a high curvature, and straight line segments have zero curvature.

Consider an 'almost straight' segment of a hypothetical river consisting of a set of equally-spaced sequential points arranged on a straight line to which has been added a slight (and potentially imperceptable) amount of random noise. Initially the curvature everywhere is essentially zero, and there is no meandering motion. However, anywhere where randomness has made the points non-colinear (as would be expected on an actual river), there will be some non-zero curvature. Furthermore, in general the random nature of the non-linearity will make the sign of the curvature change from point to point along the length (i.e. first the river will bend very slightly to the left, and then very slightly to the right, etc...). Even if this is the case only once along the river, this is enough to begin the process of instability which leads ultimately to meandering dynamics. Shown below are a sequence of X configurations of a simulated river, each Y steps apart.

<image caption='Non-colinear instability leads to meandering'>

Although the river apears quite straight to the eye initially, small random variations in curvature eventually pull sequential points in opposite directions, as each point migrates in response to the signed curvature of the river at that position. As each point moves away from its neighbors, this increases its curvature, increasing its rate of migration in a positive feedback loop. As the points move further apart, new points are inserted at equally spaced positions interpolated along the river so that the distance between points remains constant, while the total length of the river increases. Eventually the process of migration stabilizes into the familiar meandering loop pattern as the curvature along finite lengths of the river changes more gradually and becomes similar in value at neighboring points.

Therefore, once a sinusoidal channel exists, it undergoes a process during which the size and curvature of its loops increase due to the positive feedback of their motion, and then begins to slow to a near constant average rate as that feedback becomes mixed or negative due to the similarity of loop curvature at neighboring points. There are two different, but not mutually inconsistent, kinds of physical explanation for the feedback which results in meandering motion. In the first kind, the change in bend size and curvature is the result of a lateral force which causes the change in direction as the flow goes around a bend (an acceleration), and this force in turn causes a change in the non-rigid geometry of the bend wall (a stress and strain). In the second kind, any difference in the speeds of flow near the inside and outside banks of a bend causes a differential erosion or deposition of material from the bend walls, resulting in the migration of the channel.

This study makes use of meandering models of the second kind, of which the Johannesson-Parker 1989 model is the primary method in the simulations which follow. The JP89 model expresses the bend near-bank flow speeds in terms of linear perturbations of the mean value at the center of the flow, in between the banks. In the model, the rate of erosion at the outer bank of a bend is proportional to the speed perturbation at that point. If the speed of the flow near the outer bank exceeds the centerline value by an amount u1b (in the case that u1b > 0), the outer bank will erode, otherwise it will deposit material. The amount of erosion or deposition is taken to be proportional to the magnitude of u1b. Material eroded from one bank of a bend is simply assumed to be transported to and deposited on the other bank, so that both banks of the channel migrate at the same rate, and the width of the river does not change.

The JP89 paper presents an analytical model for calculating the lateral distribution of the averaged primary flow velocity (speed and direction) in meandering rivers. Their solution is actually quite complicated, and makes use of 2 coupled second-order differential or integral equations, which have subsequently been implemented in computer code by the autthors of this (MNRR) study. In terms of the mechanics of the model, the solution gives rise to a helical vortex about the midline of the flow, in the direction of the flow. This vortex creates a secondary flow in bends which is radially outward at the surface of the water, and radially inward near the river bed. This secondary flow powers a convective transport of primary flow momentum (and eroded sediment) leading to a significant outward redistribution of primary flow velocity (in other words, the flow is faster near the outer bank, and eroded sediment from the outer bank is moved to the inner bank by the vortex).

The JP89 paper attempts to accurately explain and quantify the phenomenon of redistrbution of primary momentum by secondary flow at a linear level, in the simplest possible fashion. The helical vortex is explained as a transfer of momentum from the inside of the bend to the outside. As soon as the flow enters the bend, some of its linear momentum becomes angular. As the mass of the water goes around the bend, a centripetal force is added to gravity causing the surface of the water to tilt, elevating it on the outside of the bend. This additional volume of water is provided transversely from the inside of the bend, creating a radial surface flow cross-stream. The water then moves down to replace the subsurface water pushed radially inward at the end of the bend to reestablish a level surface. The result is a helical flow, and the greater the curvature, the greater the angular momentum transformed and the stronger the cross-stream secondary current.

Thus, the JP89 model makes use of force and angular momentum to create a speed-based meandering model.

Increasing the curvature of the bend increases the force and concomitant change in angular momentum, which in turn increases the inclination of the water surface, the helical flow, and the amount of erosion occurring on the outside of the bend. This explains the positive, self-intensifying, feedback process described above: greater curvature results in more erosion of the bank, which results in greater curvature, more erosion, etc... The actual calculations of the JP89 model used in this study will be discussed in more detail in section 3.4.3.3.

The work performed on the JP89 and other models has done much to further the understanding of the meandering process, but a key question is left unanswered: that is, how important is the effect of the 3-dimensional topography of a specific river in 'steering' the evolution of meandering, in light of the success or failure of much simpler geometric or physical models? A new approach combining both the 3-d geometry and physical properties of actual river beds is called for to address otherwise unaccounted-for effects over long channel distances and migration times, where such rivers are not limited to simple bends with small curvature, and where there are complex feedback loops between flow velocity and bed topography.

Determining the essential physics required to accurately model a specific river's evolution might be considered a 'holy grail' of geophysical science. Different mathematical formulae attempt to relate the different variables of meandering geometry and dynamics in various meaningful ways, not necessarily with considerable success. In some cases, numerical parameters and functions can be established which affect the shapes of similar meandering curves in a consistent way. Nevertheless, the space- and time-varying shape of a particular meandering river depends ultimately on the geometrical and physical characteristics which can be measured from it, not on the intricacies of mathematical equations.

In order to develop a new and useful meandering model, we must first address the proper interaction of the individual elements of the model: the 3-d motion of the flow, the 3-d geometry and material properties of the meandering channel, and the 3-d topography and material properties of the landscape where they meet. However, geometry alone is not enough. For a fuller understanding of the evolution of these elements, sediment input to the channel both upstream of the flow and from the surrounding landscape due to erosion and migration of the chanel must also be addressed, as well as the deposition of that sediment on downstream banks and bed and its effect on the alteration of that geometry. However, a thorough treatment of this issue is well beyond the scope of the present work.

Good or bad mathematical models can be developed for physical phenomena, and they differ from one another in one respect by their ability to accurately represent properties of the phenomenon being investigated, and to predict observable changes in these properties over time. A good mathematical model or simulation of a river must help predict essential changes in the flowing path, the containing conduit, and the surrounding landscape, and by doing so allow human decision and intervention in order to manage resources that increase the benefit that the river carries as a source not only of commerce, but of the lives of the people, animals and vegetation around it.

2.3 Simulation strategy used in this study

The simple meandering models typically used to simulate the shape and motion of rivers cannot generate the specific detail which is unique to the Missouri river. Although the Johannesson-Parker 1989 model is considered to be the 'state of the art' in this field, it cannot simulate the multiple channels, the 2-dimensional 'braiding' patterns, nor the many chutes, islands, and sand bars (both submerged and above the water surface) which are integral features of the 59-mile segment of the MNRR. However, by performing a large number (3000+) of simulations initialized to the shape of the current river, and each using slightly different physical input parameters, we can generate a diversity of river shapes and dynamics that --although different from one another and from the precise shape of the actual river--

will, taken together, approximate what we think the river is likely to do during the next 100 years.

These aggregate results, although completely deterministic in nature, provide a probabilistic interpretation of what the single actual river might do, and where it might go, during 100 years. 'Target' areas within the river valley can be defined which correspond to towns, roads, airstrips, parks, and other geographic landmarks. The aggregate 100 year 2-dimensional coverage array can then be overlayed on these targets to see which of them are encroached upon by the river, to what degree, and when. Specifically, the 2-d coverage array indicates what fraction of all simulated rivers passes through any point (represented as 125 x 125 m square areas) within the valley at some time during the simulation. This array is calculated every 10 years, so that the potential progression of the river across the valley can be tracked over time.

A database was programmed to make use of information saved with each run of the simulation in order to track the course of every individual river over time to see when, and how much area of, each target it will eventually reach and subsume. The database can also find examples of individual simulated rivers which reach and flood several designated targets at some time during the simulation, but which miss others. This information, along with the 2d coverage maps discussed above, can be used to assess the relative danger from the river to specific locations within the valley at different times during the next 100 years. In addition, contour plots showing the percentage (e.g. 95%, 75%, 50%, 25%, 5%, 1%) of all simulated rivers reaching points in the valley indicate the relative danger of being overrun by the river during 100 years to towns and other regions lying on or inside these lines.

3072 different simulations were performed using the Johannesson-Parker 1989 meandering model. An additional 540 simulations were performed using a simpler model as a comparison. All 3612 simulations amassed over 16 Gb of 'raw' data, composed of individual binary files storing river shape coordinates, 2d coverage arrays, cumulative erosion, and other information at 10 year intervals, as well as images of each final river. Any piece of information from this data set can be retrieved and used for analysis by the database program, and additional images and image mosaics of the rivers can be created as desired. Analyses of these results contribute over 155 Mb of additional data. Software and tests performed previous to the full set of simulations bring the total project size to over 17 Gb. Some of this data is available online at the website listed below.

2.4 Link to companion website

This report, a 3-page summary of the project, 33 detailed project updates, a software archive, and selected data sets, are available online at:

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http://www.skycoyote.com/sky/MNRR/
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This site contains many additional images and calculation details, and the summary pages are suited to a non-technical audience who is not familiar with this research.

3.0 Approach & methods

The overall approach used in this study consists of the following steps, each of which will be discussed in more detail in the sections below:

1. Create mosaics manually pieced together from several USGS topographic maps and Google Earth satellite images which cover the 59-mile segment of the Missouri National Recreational River and surrounding valley from Gavins Point Dam (near Yankton, South Dakota) to Kensler's Bend (near Ponca, Nebraska).
2. Digitize the longitude and latitude coordinates of the left and right banks of the river, and the valley walls at the 1200' topographic line, at 100 m intervals.
3. Calculate the river centerline and width at 100 m intervals from the digitized left and right banks.
4. Convert the longitude and latitude coordinates to isotropic x and y coordinates based on a zero origin at the Gavins Point inlet using oblate ellipsiod geometry and multiple tangential planes.
5. Generate a set of input parameter combinations of physical values intended to yield a range of realistic behavior in the simulated river using the JP89 meandering model.
6. Run many 'constrained' river simulations using the valley boundaries and river inlet endpoint as fixed positions which limit the motion of the river.
7. Accumulate quantitative statistics for erosion rate, total area eroded, occupied area (2d coverage arrays) of valley, and occupation duration for {x, y} points in the valley, for all simulations.
8. Combine valley coverage arrays from all simulations, and from subsets of simulations derived from specific combinations of input parameters, to create a 'common coverage' array which delineates the '100 year migration corridor' of the aggregate river at from 95% to 1% confidence levels.
9. Overlay the common coverage array and contours on maps and satellite images of the study area.
10. Designate 'target' areas in the valley, and calculate the fraction of simulations which reach and pass these areas, and their times of acquisition during the course of the simulation.

3.1 Computer software used

All of the computer software created and used in this study was written in the Python language, version 2.5.2, which is available free of charge for most computer systems at www.python.org. Python is a high-level cross-platform object-oriented programming language which is run by an interpreter program usually written in the C programming language. Python is well known for its ease of use and rapid development time. In the opinion of the authors of this study, all new scientific software should be written using Python, if possible. Also used were the WxPython 2.8.8.1 (www.wxpython.org) and NumPy 1.1.1 (www.numpy.org) libraries for graphics/user-interface and vector numerical calculations, respectively. Both libraries are also available free of charge for most computer systems.

All software was developed and run under the Apple Macintosh OS X 10.5.5 'Leopard' operating system. Development was performed, and simulations were run, on an Apple iMac 2.4 GHz Intel Core-2 Duo with 4 Gb of Random Access Memory and on an Apple MacBook 2.0 GHz Intel Core-2 Duo with 2 Gb of RAM. However, all software should run fine, with no or only slight modifications (e.g. for binary file access), on any Unix or other operating system which supports Python.

3.2 Maps & satellite images used

A single mosaic was made from 24 1957-1994 USGS 7.5' topographic maps which were individually downloaded from store.usgs.gov. These maps were rasterized from PDF, rotated by from 1.1 to 1.65 degrees clockwise, layered on a single canvas, and aligned by hand in Photoshop. The overall map projection was polyconic. This mosaic contains sufficient detail for digitizing the river banks to better than 10 meter resolution, although we have extracted coordinates only every 100 m.

<image caption='USGS map mosaic'>

A second mosaic was made from 17 Google Earth sattelite images screen-captured at an 'altitude' of 20.17 km from earth.google.com using their free client program. These images were also assembled by hand in Photoshop. Although nothing is known about the actual projection used in these images, it is a reasonable assumption that they were also rectified individually via digital image processing to conic projections depicting a stationary 'birds-eye point of view'.

<image caption='Google Earth satellite mosaic'>

By interpolating the satellite mosaic by 2x, and then independently adjusting the x and y scales to 103.5% and 103.05% respectively, the two mosaics can be overlayed and aligned at the same scale, even though the topographic maps were made at different dates from 1957-1996, and the satellite images were made much later (2006). Although both image sets were assembled by hand, the correspondence at both the Gavins Point Dam end of the river and the Sioux City end are quite good, and the alignment is virtually exact near the center of the mosaics. Since this provides a resolution of about 8.99 m/pixel (for the mosaic, a dx of 68143.17 m and a dh of 7579 pixels) and a visual error of at most a few pixels, we feel that this alignment is sufficient for the 100 m discretization used in this study (as will also be shown in the next section). Although the position of the river can be seen to have changed in some places between the time the two image sets were acquired, the endpoints and other man-made features are the same in both.

<image caption='Superimposed mosaics'>

<image caption='Close-up of Gavins Pt end'>

<image caption='Close-up of Sioux City end'>

3.3 Computer programs created for this study -- 3.3.1 Digitization of river coordinates

A simple Geographic Information System (GIS) program was written to semi-automatically digitize the coordinates of the river banks and valley walls from the map mosaic shown above, and to interpolate these coordinate paths to a resolution of 100 m using circular arcs (see section 3.4.4.1 below for details). The meandering routines described in section 3.4.3 below require a river centerline and width at all points as input, and the GIS program computes these based on the digitized bank coordinates (see section 3.4.1.2 below for details).

<image caption='River digitization program'>

This program is easy to use, and produces good results quickly. Program features include:

* A background image can be displayed as a guide.

* Points can be entered with the mouse on multiple curves.

* Points can be added, inserted, selected, moved (by mouse or keys), or deleted. Curves can be joined, split, or reversed.

* The current session can be saved to a file and restored so that digitization can continue at another time.

* All coordinates are in longitude and latitude decimal degrees. Coordinates reference landmarks (red stars in image shown above; at least 2 required) which can be entered using known map positions.

* The display shows the longitude and latitude of the window center, and the scale (meters/pixel) at that point.

* Distances are calculated using oblate ellipsoid geometry, where:

<image caption='Distance calculations'>

and a = equatorial radius, b = polar radius, phi = latitude, lambda = longitude. (See http://en.wikipedia.org/wiki/Earth_radius for details)

* Coordinates can be printed and used as input in meandering simulations:

<table id=13 caption='Digitized river coordinates'>

* The program has unlimited undo and redo capability. The following operations automatically save a copy of the current data and settings in memory:

1. Adding a point,
2. Deleting points,
3. Moving points,
4. Joining paths,
5. Splitting a path,
6. Reversing a path,
7. Moving the background and points,
8. Zooming in or out,
9. Loading a new background.

Only the first of several contiguous additions, moves, or zooms is saved. In addition, the current state can be manually saved with a button at any time. Previous states can be visited by clicking another button, which in turn saves 'future' states for redoing, if desired. By saving all intermediate states, one can undo operations back to the beginning of the session, and then redo operations up to the end again. Past and future states are saved with the session file on disk, as well.

* Path points can be resampled and interpolated at a designated resolution using circular arcs. After interpolation, points are evenly spaced and are placed on piecewise circular arcs (although distance is still calculated linearly). This has the effect of smoothing the curve, of ensuring that every point lies on a tangent with a well-defined radius of curvature, and of maintaining a more continuous change of curvature along a path. Here is an example of before and after interpolation:

<image caption='Coordinate interpolation'>

* Curve attributes (name, colors, visibility, etc...) can be edited.

* The display can be zoomed in and out, and the background image and all points are rescaled accordingly.

This program was used to perform a digitization of the MNRR and surrounding valley from USGS topographic maps combined as described above. River bank cordinates were entered by hand (with the mouse) about every 125-250 m, and then interpolated to 100 m. The valley boundary was digitized at about half this resolution along the 1200' topographic line, and was then also interpolated to 100 m.

Here is the resulting digitized river and valley superimposed on a 1 km grid:

<image caption='River valley coordinates'>

Here is detail in the mid-section:

<image caption='River valley detail'>

Note that:

* The river banks were digitized from Gavins Point Dam to just below the city of Ponca, where the river begins to be bounded by concrete walls and wing dikes. The resulting lengths of the right and left banks were 98.748 km and 101.541 km respectively.
* The valley 'walls' were digitized wherever obvious. Otherwise, the 1200' topographic line was followed (although not in as much detail as the river banks). This line was cut across intersecting canyons entering the valley, rather than followed up into them.
* The valley is fairly flat throughout. The river falls from about 1170' at Gavins Point to about 1100' below Ponca. Mid-river is at about 1140'. The 1200' elevation line was considered to be a likely boundary of the meandering river.

We have attempted to capture a 'reasonable' rather than 'exact' facsimile of the river from these maps, and have been constrained by the meandering models to digitize a single channel throughout the extent of the river, and a single left and right bank. Therefore, while we have not included chutes or other bifurcations in the main flow, we have included several islands.

<image caption='Omitted details in digitization'>

Ideally a later version of the meandering program can properly take these into account (see section 5.4.3.1 on multiple threads). We have also attempted to combine conflicting bank data from maps made at different times (most notably the Burbank/Ponca maps from 1994 and the Elk Point map from 1963):

<image caption='Conflicts in digitization'>

In all cases, we have attempted to render a reasonable-looking whole, rather than favoring any particular time period over another. Nevertheless, superimposing the resulting digitized banks over a satellite mosaic image indicates a good correspondence between the maps, digitized banks, and satellite images, although some features present in the maps no longer exist, and the river has clearly migrated during the intervening time:

<image caption='Digitized river on satellite mosaic'>

Here are details of the (fixed) end-sections of the digitized river superimposed on the satellite mosaic:

<image caption='Details of superimposed ends'>

3.3.2 Tests of meandering methods

Another program was written to simulate and display the evolution of a meandering river as it migrates, using one of several different models. The program also accumulates the cutoffs which occur as the river loops back over itself. This program provides a convenient way to test new meandering models and observe results immediately. Physical parameters can be entered into the graphical user interface of a control window, and results can be viewed in a changing plot window as the model is running. The meandering calculations are performed in a separate thread of execution so that the user interface and plot animation does not block. The meandering code interface is modular, so that different models can be 'swapped in' as required without editing the rest of the code. The main program calls the meandering module, provides for udating and displaying the river coordinates, and for automatically interpolating new river points as they are required.

<image caption='Stand-alone meandering program'>

The changing coordinates of river points, and data exchanged with the meandering routines can also be displayed on the terminal:

<table id=4>

One interesting feature of the program is that the interpolation of new river points is performed using circular arcs, rather than straight line segments, so as to preserve the curvature of the stream in new points (see section 3.4.1.1 for details). Thus new points are created with a curvature similar to the surrounding points (rather than zero), and therefore the curvature does not increase erroneously at existing points. Another nice feature is that the state of the simulation is automatically saved at periodic intervals or whever it is paused. Any number of intermediate states can be accumulated, and it is possible to go back to any previous state by clicking a button. The simulation can then be resumed from a past state (possibly using new input parameters), or can be stepped forward again without having to recompute the intermediate points.

This program has been used to test both the JP89 meandering model and other, simpler, models, as described in detail in section 3.4.3 below. The outcome of these tests indicate that it is both possible and fairly easy to generate the characteristic meandering looped shape of a river (e.g. 'Kinoshita curves'), including the upstream asymmetry of their lobes, using a broad variety of methods and parameters, and not just from a complex physical model such as JP89. All models are derived from the curvature of the river, either 'local': the inverse radius of a circle passing through a point and its two nearest enighbors, or 'effective': the integral of local curvature along a finite segment of the river, usually upstream of the point to which it applies. Meandering models can be based on an algebraic equation of either force or speed, or on differential or integral equations (as is JP89). Nevertheless, all can produce similar general qualitative behavior.

The program has the ability to accumulate excised cutoffs and display them in the plot. Cutoffs can be kept and 'aged' for a certain number of additional steps and then discarded, or kept and displayed forever. Note that the cutoff plot does not necessarily show the full meandering range of the river, as the entire history of the main stream is not displayed, only the current stream and all past cutoffs. Nevertheless, the main stream and accumulated cutoffs give a good indication of the extent of the planform at any time.

Here is a montage made of a simulation showing all cutoffs during its evolution starting from a nearly straight line with slight x and y perturbations over the course of 17 intervals of about 100 steps each:

<image caption='Evolution of simulated river and cutoffs'>

3.3.3 Simulation of a single river

The main program used in this study was written specifically to perform a single simulation of the 59-mile segment of the MNRR, although it could in principle be initialized by an appropriate input file to simulate a segment of any other single-channel river which has been digitized by the GIS program described above. This program reads a text file of inputs describing the shape of the river (x and y coordinates of the centerline, and width of the river at each point), and several name/value pairs of specific input parameters which control the simulation.

<table caption='MNRR simulation input file'>

<image caption='Initialized MNRR simulation'>

Features of this program include:

* The valley boundary, river coordinates, centerline, or filled width can be displayed on a background consisting of a 1 x 1 km grid or a selected image. The 2d coverage array, residence times, or cumulative erosion (see section 3.4.6 and 3.4.7 for details) can be superimposed on the coordinates and background.
* The simulation can be saved to a binary file, reloaded, and continued at a later time.
* The coordinates of river points can be printed as text.
* The simulation can be run for any number of steps, saving 'snapshots' of the simulation at regular intervals for review. The simulation can be returned to any saved state and animated through a sequence of states either forward or backward, allowed to run forward with a new set of input parameters, or reinitialized from an external text file of coordinates and parameters.
* The simulation can make use of a number of different modular meandering models. Details of the data sent to and from the meandering module can be printed as a diagnostic.
* During the simulation, cutoffs can be detected and removed, saved and displayed, or discarded. The river coordinates can be checked against the valley boundary and maintained within the designated domain (see section 3.4.1.4 for details).
* The erosion rate can be calculated at each step, and the migration rate can be automatically adjusted to initialize the erosion rate to a specific value (see section 3.4.5 for details).
* Physical parameters constraining the simulation can be changed from the user interface, or imported from parameter files.

Default parameter values were set based on the following sources of information:

* A constant flow was estimated by averaging the Gavins Point Dam discharge quotes from Table 2 of [ref xxx]. This value is consistent with the discharge plots in Figures 2 & 4. This value will be varied as discussed in section 3.3.4 below.
* A constant depth was estimated from gauge height plots at Yankton and Sioux City, and corroborated by the National Park Service website. This value will also be varied as discussed in section 3.3.4 below.
* The bed particle size was set at 1 mm, consistent with a medium sand basin. This value will also be varied as discussed in section 3.3.4 below.
* The water surface slope was calculated from the difference in the inlet elevation and the outlet elevation, divided by

the current length of the river. The inlet elevation was estimated at 354.76 m, and the outlet elevation at 336.40 m, based on digital elevation data downloaded from the National Map website.

* The lag distance and sum distance (see section 3.4.2.1 below) were set to integrate local curvature along an interval of one river width, one river width upstream from each meandering point. These values will also be varied as discussed in section 3.3.4 below.

* The river migration rate was adjusted so that the initial total erosion rate estimate (using 'unmeandered' river geometry) was close to 1.2 ha/yr/km, as quoted in Table 1 of [ref xxx]. The initial erosion rate (and therefore the migration rate) will also be varied as discussed in section 3.3.4 below.

Two important caveats concerning the parameters and constraints of the simulation should be mentioned. As is discussed in section 3.4.1.5 below, the maximum width of the river in most simulations (and in all JP89 simulations) was fixed at one of several constant values (from 500-800 m) along its entire length from the inlet at Gavins Point Dam to the outlet at Kensler's Bend. In addition, while the inlet point of the river was fixed in place at all times during the simulation, the outlet was allowed to move freely (in accordance with the meandering model) as if the concrete walls and wing dikes below Ponca did not exist (see section 5.3 for discussion).

Here is an example of a 100 year evolution, shown every 10 years, of a simulation initilized with the digitized MNRR coordinates, using the XXX meandering model, and physical parameters:

<table caption='Simulation parameters'>

<image caption='Simulation evolution'>

Here are the final 100 year configurations of a few 'constrained' simulations (i.e. the inlet has been fixed in place and the river cannot exceed the boundary). These test simulations show far more activity than is expected of the real river over the same time period, by virtue of the initial erosion rate being set far larger than has been measured for the actual river:

<image caption='Test simulations'>

The constrained river simulations can also be overlayed on a map or satellite mosaic:

<image caption='Simulations on background'>

Different combinations of the various simulation parameters (e.g. width, depth, flow, bed particle size, integration distances, etc...) yield an enormous variation in river shape. Some of the configurations are quite 'wild':

<image caption='Unrealistic simulations'>

For this study, however, we are interested in more realistic simulations that are fairly 'tame', and have erosion rates commensurate with measured values:

<image caption='More realistic simulations'>

If these more realistic simulations are interpreted as the behavior of a 'median river', then a range of input values which include those which produce these simulations could be expected to generate a population of rivers whose shapes and dynamics were representative of what was expected of the actual river. This is the topic of the next section.

3.3.4 Multiple simulations framework

The strategy used in this study is based on the assumption, confirmed by observation, that none of the meandering models or input parameter combinations will correctly genrate the geometry and dynamics which are specific to the Missouri river. This is trivially seen to be true since all of the current river models used simulate only a single channel, while the Missouri contains numerous bifurcations, chutes, islands, braids, and submerged and surface bars, making it an especially challenging subject to model. Therefore, an intelligent choice of many slightly different simulations, utilizing a carefully selected range of input values, is expected to better approximate, as a population, the likely behavior of the single real river over the next 100 years.

The selection of parameter choices to be used will be discussed in section 3.4.8. The ability to automatically perform mutiple simulations and accumulate planform/migration statistics is an enhancement of the single-simulation program discussed above. This program was modified to read and parse text files containing commands instructing the program to:

* Load a base model of river coordinates.

* Reset values of specific simulation parameters.

* Automaticaly vary migration rate to initialize erosion rate to a specified value.

* Run a 100 year simulation.

* Save the results to a binary file.

* Save an image of the final river configuration.

* Repeat the steps above for any number of different simulations.

Here is an example of a partial command file used:

<table caption='Multiple simulation command file'>

This automation capability makes it possible to perform simulations spanning an n-dimensional grid of input parameter space in order to search for likely combination candidates (see section 3.4.8 below). Each 100 year simulation can take anywhere from several minutes to several hours to complete, depending on the resolution of the dt step size and interpolation distances used. Final river geometries may be reviewed manually or semi-automatically (see section 3.3.6 below) after all runs have been completed. Subsets of results which appear promising can then be used to refine the parameter search. This iterative process was used to find parameter ranges used for the large set of runs performed in this study.

Test runs to select combinations of input parameters usually took about 1-2 hours to perform, while the final runs used in this study took about 15-20 minutes each, using a dt of 0.25 years. While the total continuous time required for these runs (n = 3612) was in excess of 43 days, this was ameliorated by two factors:

1. Simulations were broken into 3 groups and run simultaneously on 2 different computers (and in parallel on dual-core processors).
2. The command file syntax has a 'goto' command that allows a batch run to be stopped and resumed with a particular simulation at a later time.

Even so, completion of all simulations running 10 hours per day took about 35 days.

3.3.5 Graphics

All output graphics and user interface input were performed using the free WxPython cross-platform library (www.wxpython.org). Graphics were an integral part of all programs, rather than an additional post-processing step, so that it was possible to view the intermediate results of all simulations and other calculations in real time as they occurred, and to take immediate user action based on the evaluation of that output (e.g. to stop the simulation or change parameters and continue) without waiting for a long-running program to conclude. It would not have been possible to perform this study in the time period available without this interaction capability.

3.3.6 River database

The final major program to be written for this study was a 2-part database which was used to accumulate information about all simulations as a post-processing step, and to display and manipulate these results in both numerical and graphical form. This database has a programming and query syntax similar to the 'Structured Query Language' family of popular programs (e.g. 'MySQL' and 'PostgreSQL'), so we have elected to call it 'RiverSquirrel'.

First, a database creation program peruses all completed simulation files and accumulates several numerical results in 1- and 2-dimensions (including length, number and length of cutoffs, sinuousity, erosion rate, coverage area and residence times, cumulative erosion, etc...), creating a large table of values. Then, a second database query program can perform the following operations on this table:

* Print entries and fields in any order.

* Sort entries about any field.

* Select subsets of entries using numerical relationships and combinations of 'and' and 'or' clauses.

* Plot the final or intermediate river for any input parameter combination.

* Accumulate and plot common coverage and average erosion as 2d arrays for subsets of entries.

* Plot mosaics of river coordinates, background image, and 2d arrays and contour lines as transparent overlays.

Individual queries (e.g. selecting and sorting) can also be chained together to form more complex compositions. Once the database creation step has been performed, all query operations are quite fast to complete. Currently the query interface uses a programming interface from Python, but it could easily be modified to read prepared text files, or to take interactive input from the terminal or over a web connection.

The database can track the course of every individual river over time, and can also extract specific examples of river behavior (such as the greatest and least lengths, coverage, or erosion rates, or rivers which pass through specific points in the valley or other designated targets at some time during the simulation), and can plot histograms and trends in the behavior indicated by relationships between 2 or more input or output variables. See sections 4.2 to 4.6 below for many examples of using this database to extract, analyze, and display simulation results.

3.4 Calculations -- 3.4.1 Discretization and interpolation -- 3.4.1.1 Interpolation via circular arcs

Both the river digitization program and the river simulation program represent curves (e.g. river centerline) as paths parameterized by length, where intermediate points are placed on circular arcs rather than on line segments. What this means is that:

* A unique circle having a specific {x, y} center and radius is calculated to fit every three consecutive {x, y} points along each curve, and
* New points added to the curve are placed somewhere on the perimeter of these circular arcs, rather than on straight lines connecting existing points, so that
* As the length of a curve grows, or the distance between points is changed (as is the case when the simulated river meanders and grows in length, or when a newly digitized curve is interpolated to equal distances), points are calculated and placed at equal distances along the total length of these circular arcs. Linear interpolation is used only if a set of three consecutive points is colinear (i.e. fits a circle of infinite radius).

Using circular arcs, rather than straight line segments, has the effect of preserving the curvature of the path in new points. New points are created with a curvature similar to their surrounding points (rather than zero, as would be the case with linear interpolation), and therefore the curvature does not increase erroneously at the existing points. As an example, consider a stream consisting of a single circular arc. If, as the stream lengthens, points are interpolated using straight line segments, the stream departs from circularity and existing points along the perimeter accumulate excess curvature:

<image caption='Growing circular path with linear interpolation'>

If circular arc interpolation is used, the circularity and constant curvature at each point is preserved:

<image caption='Growing circular path with arc interpolation'>

In cases where the center or radius of arcs change along the path length of the curve, new points are placed using a weighted average of the previous and the next arc in the sequence, where the weights of the average are based on the fractional distance of the new point along both arcs (e.g. a new point to be placed at a distance exactly between two existing points i and i + 1 would be placed at the average of 2 points, one on each of the arcs fitting points {i - 1, i, i + 1} and {i, i + 1, i + 2}).

Although all programs created for this study make use of circular arcs in their calculations, the graphics used to display points on curves is still based on linear interpolation, as the difference between the two representations is usually not discernable to the eye.

3.4.1.2 Curvature and effective curvature

All of the meandering models described in section 3.4.3 below depend on the curvature at each point on the simulated river as their primary independent variable. Often, curvature is calculated from the vector differences between successive points in the river. In this calculation two vectors u and v are formed at each point $p(i) = \{x(i), y(i)\}$ such that $u = (p(i) - p(i - 1)) / |p(i) - p(i - 1)|$ and $v = (p(i + 1) - p(i)) / |p(i + 1) - p(i)|$, where || denotes vector magnitude. The angle between the two vectors is given by the arcsine of their cross product, and the radius of curvature is then calculated from this angle and the average distance between the points p(i - 1), p(i), and p(i + 1).

In this study, however, curvature is given directly as the inverse of the radius of the circular arc passing through each point, as described in the previous section. This has the effect of avoiding the spurious overestimate of curvature which would otherwise be calculated by the vector method at places where the river changes direction suddenly, especially at the intersections of loop cutoffs. The abrupt change in direction at excised cutoffs and at boundaries which limit the motion of the river would generate excessive curvature which could in turn cause discontinuities in the meandering motion. By the use of circular arcs passing through every three sequential points in the river, this anomalous behavior can be avoided.

<image caption='Excessive curvature at sharp bend in river'>

<image caption='Curvature limited by circular arc'>

In addition to the local curvature C(i) calculated at every point in the river, the meandering routines described below all make use of the 'effective curvature', which is simply an integral of the local curvature over some finite interval in the river, usualy upstream. The location and the length of this integral is set by the two input variables ldist ('lag distance') and sdist ('sum distance'), which are given in terms of the width of the river at the point to which they apply. These variables reflect the hypothesis that the migration of a specific point in the river is partially an effect of the geometry of the river upstream from that point. The variables ldist and sdist control how far upstream, and over what interval, the local curvature is averaged to calculate the effective curvature used in the meandering models. Nominal values for both variables are 1 river width. By slightly changing these values from unity, both the size and shape of loops formed by the migration of the river can be altered, including the upstream or downstream asymmetry of those loops.

3.4.1.3 Centerline and width

Although the river has been digitized from the map mosaic as two curves representing the left and right banks, the meandering models require a single centerline and width as their input, and the migration displacements produced by these models are applied to points on that centerline. Therefore, it is necessary to calculate a centerline from the left and right banks, and to estimate the width of the river at each point on that centerline in a direction which is perpendiciular to the downstream direction.

This is done automatically by the GIS digitization program after the left and right banks have been entered and interpolated on arcs as described above. In this case, the 'downstream' direction is defined as that which is tangential to the arc passing through each point on the resulting centerline, while the perpendicular 'cross-stream' direction is parallel to the radius of that arc. Thus, an additional benefit of using circular arc interpolation is that down- and cross-stream directions can be uniquely and robustly defined at each point on the river. The result is an 'idealized' river geometry that closely follows the actual river, but which can be described by a single smoothly varying curve and width at every downstream point.

The algorithm used to calculate the centerline is computationally intensive (requiring slightly less than 5 minutes for two 100 km banks), but seems to work well for all parts of the river, including areas where the banks change direction and/or are highly curved. Although there are probably many ways that a centerline and width might be approximated, the algorithm used in this study is based on an iterative procedure which starts from a small set of corresponding pairs of points on both banks, and repeatedly adds additional pairs of points until all candidates have been exhausted. The basis of this calculation is the following definition of 'width', which can be taken as an axiom only for a certain set of points which meet the following criterion:

* The 'width' of the river is only well-defined for those corresponding points {L(i), R(j)} along the left and right banks L and R such that:

* For a given point L(i) on bank 1, the closest point on bank 2 is R(j), and for that point R(j) on bank 2, the closest point on bank 1 is also L(i). The width of the river at {L(i), R(j)} is then the distance between the two points, and the centerline is defined to pass halfway between them.

One might, in principle, imagine two surveyors walking along the left and right banks of the river, each holding an end of a rope which is stretched across the river between them. As they move along the banks, sometimes the rope can be pulled tight and short between them, and sometimes it must be much longer, depending on both of their positions on the banks, and the actual width of the river. If the surveyor on the left bank stays at the same position while her partner on the right bank moves back and forth along the strand, they will discover a configuration where the rope is shortest between them. Similarly, if the surveyor on the right bank stays at the same position while her partner on the left bank moves back and forth along the strand, they will discover another configuration where the rope is shortest between them (and not necessarily the same configuration as before).

The constraint stated above says that if the two configurations just described are the same (both surveyors in the same positions when the rope is shortest from either side), then they have found a place where they can without doubt define the width of the river. Put another way: given the positions of the two surveyors, if the person on the left bank moves by any amount in either direction, the rope will by necessity become longer, and simultaneously, if the person on the right bank moves by any amount in either direction the rope will also become longer, then they have found a stable minimum distance between the two banks with respect to downstream distance, and can therefore uniquely define the width of the river at that point.

<image caption='Minimum distance across river with repsect to bank motion'>

Application of this constraint yields surprisingly few mutually corresponding points along both banks, as in general the closest point R' to point L has as its closest point a different point L' $\neq$ L. However, this criterion can be applied iteratively, beginning with the set of corresponding points which are closest to one other, and then adding additional pairs of points in order of greater global distance between them such that:

1. New {L(i), R(j)} combinations are selected as above from remaining points after all previous {L(k), R(l)} pairs are excluded, and
2. A new pair {L(i), R(j)} is only added if the line segment connecting L(i) and R(j) does not cross any previous line segments connecting already existing L(k) and R(l).

In terms of the two hypothetical surveyors: as they move along the left and right banks and discover mutually minimal points as described above, they place stakes in the banks at their current positions and stretch ropes across them. Once they have found all such pairs of points, they begin to re-examine the points which are in between the points they have already found. In doing so, they look for new pairs of points which are mutually closest togther, such that in making their new measurements they do not move so far as to pass any already staked-out points (although they may re-use existing points paired with new points). As they find new corresponding pairs, they plant new stakes and stretch new ropes across them, such that none of the ropes intersect mid-river (although some stakes may have more than one rope attached). Eventually, they will have exhausted all possible pairs of points. The centerline is then defined as the midpoint of each rope, and the width at that point is half the rope length.

This process is not necessarily one-to-one, as a point on one bank may be connected to several points on the opposite bank, as long as the two specified conditions hold. And, some points may not be connected at all, if they fail the conditions. In general, however, nearly all points on both banks will be connected to at least one other point, so that the resulting centerline is well represented. Nevertheless, some smoothing and reinterpolation of the centerline is useful as a post-processing step. Here is an example of this process applied to a subset of the MNRR:

<image caption='Digitized and interpolated banks'>

<image caption='Calculated and interpolated centerline'>

<image caption='Cross-stream widths'>

<image caption='Widths filled by circular graphics pen'>

The centerline coordinates and widths can be printed to the terminal or a file, to be used as input for the meandering simulations. The banks, valley walls, and any other curves can also be printed. Output is in {longitude, latitude, path length, width}):

<table caption='Centerline and width coordinates'>

3.4.1.4 Conversion to {x, y} coordinates

The river and valley coordinates digitized from the map mosaic are in terms of longitude and latitude angles on the surface of an oblate ellipsoid. However, the meandering model code requires inputs in terms of x and y coordinates in meters which are part of the same rectangular planar surface. In general, it is not possible to perform this transformation uniquely, although it is possible to approximate it for sufficiently small planes which are tangent to the ellipsoid surface at discrete points of interest. Since the river is represented by a 1-dimensional curve which extends primarily in a west-to-east direction from its inlet to its outlet, this approximation can be performed at each point on the river centerline without too much loss of accuracy.

A small utility program was written to convert the river {longitude, latitude} coordinates into {x, y} coordinates that are part of the same plane. Although this is not a complicated calculation, it is somewhat subtle and merits a little discussion. The MNRR bank coordinates, and the calculated centerline, were digitized as {longitude, latitude} pairs from USGS topographic maps. These maps were made using polyconic projections, and are themselves nearly equivalent to tangential planes at the center of the projection. During the digitization process, points were converted from screen coordinates to longitude/latitude using two landmark reference points at {-97d27m30s, 42d55m} and {-96d35m, 42d32m30s} which straddle the river in both north/south and west/east directions. Longitude and latitude of digitized points were calculated by bi-linear interpolation between the two landmarks using separate scales for longitude and latitude that were assumed to be constant across their bounding rectangle. Although this is not strictly correct, it is sufficiently accurate for use in this study, as will be shown below. Ideally, several landmarks along the path of the river should be used, with local longitude and latitude interpolated from the closest of these for each point on the river.

The river simulation needs x and y coordinates in meters, and all must reference the same origin (arbitrarily defined as the inlet at Gavins Point Dam). This is impossible for points on an oblate spheroid which are not all on the same geodesic, especially as the length of the river is over 95 km. Nevertheless, a good approximation can be performed using the small-distance calculation:

<image caption='Distance calculations'>

where a = equatorial radius, b = polar radius, phi = latitude, lambda = longitude, and one or more small XY planes tangential to the surface of the Earth. The range of longitude and latitude from the beginning to the end of the river is 0.83 degrees and 0.30 degrees respectively, so the distortion incurred by this approximation isn't to great. However, it is not isomorphic, due to the difference in radius between a circle of longitude (great circle) and a circle of latitude (not a geodesic) at the latitude of the river (about 43 degrees).

We transformed coordinates to x and y first using a single tangent plane at the midpont in longitude and latitude at {-97.064, 42.713} degrees using two separate scales for converting delta-longitude to delta-x and delta-latitude to delta-y, measured from small changes in these angles at that point. Angular differences of 1 micro-radian at this point produce linear distances of 4.694 and 6.365 meters respectively, yielding scales of 81919.690 and 111087.144 m/deg. Calculating x and y for the river using this constant scale yields values from {0.0, 0.0} at the inlet to {68199.458, -32186.265} at the outlet and a total length of 95402.556 m, which is close to the 'actual' centerline length of 95353.747 m given by the GIS program.

We also transformed to x and y using different scales calculated at centerline points along the length of the river, which correspond to using a set of many small XY tangent planes, each situated on the centerline at the midpoints between each of the 100 m digitized points. Each set of scales was calculated as above, and the pair for pts i and i+1 were used to calculate the delta-x and delta-y bewteen those two points, from the beginning to the end of the river. Transforming in this way yields values from {0.0, 0.0} to {68143.168, -32186.223} and a total length of 95353.747 m, which is 'exact' to within less than 1 mm. The final x and y values for the single-plane vs. multiple-plane transform differ by about 56 m in x and 4 cm in y.

As a comparison, we also performed a transform using an isomorpic scale in x and y derived from the Euclidean distance in longitude and latitude between the river inlet and outlet (as if the river was on a flat surface and delta-lon = c * delta-x and delta-lat = c * delta-y for some constant c). All 3 methods are plotted below, where red = isomorphic, green = single plane at midpoint, blue = multiple planes along centerline:

<image caption='3 coordinate transforms'>

<image caption='Detail at outlet of river for single vs. multiple planes'>

3.4.1.5 Boundary checks

The boundary of the MNRR valley was digitized at approximately the 1200' topographic line. The simulated river is constrained to remain within this boundary, and to approach it no closer than 1/2 the width of the river. The location of the inlet point at Gavins Point Dam is also fixed in place, as are points on the first half kilometer of the river. Points on the next 800 m of the river have their motion damped by a negative power of 2 in the sequence {1/256, 1/128, 1/64, 1/32, 1/16, 1/8, 1/4, 1/2} multiplied by the displacement generated by the meandering model.

The downstream end of the river is free to move in accordance with the output of the meandering model, although in reality the tail of the river is constrained by the artificial channel and wing dikes of Kensler's Bend. This was done so that the length and shape of the simulated river could grow without limits other than those imposed by the topographic boundary, and that therefore the 'migration corridor' defined by that motion would represent an 'upper bound' to that of the river constrained at both ends. Tests performed with the outlet fixed in place did not differ substantially from those in which it was free to move, and so the decision was made to always permit this motion.

The boundary forms a closed irregular polygon around the river. All river points are checked after each migration step to ensure that:

1. They are within the boundary.
2. They are at least 1/2 river width from all boundary points and edges.

Points which are in violation of either of these constraints are moved according to the following procedures:

1. Points too close to but inside the boundary are moved away from the boundary along a vector perpendicular to the nearest edge until they are 1/2 river width away.
2. Points which have moved outside the boundary are moved inside along a vector perpendicular to the nearest edge until they are 1/2 river width away.

Thus, the boundary forms a 'hard' limit to the motion of the river, and does not otherwise affect the migration rate. This process allows the river banks to become very flush along the boundary perimeter, respecting the current river width. The boundary was originally digitized at a resolution of 100 m, but that was found to make the boundary-checking code too slow (and, most of the simulation program time was spent in boundary checking each step). The boundary resolution was subsequently reduced to 1 km per edge, which is sufficient for containing the river geometry (which is digitized at 100 m) within the valley, but is fast enough for practical purposes.

3.4.1.6 River width

After having digitized the MNRR and performed preliminary tests with the meandering models, we have observed the following:

1. The digitized river has a minimum width of 186.79 m, maximum of 1647.97 m, mean of 747.84 m, and standard deviation of 337.62 m. We do not believe this to be an accurate representation of the active meandering width of the flow, and in general believe that the observed bank-to-bank surface width is a serious over-estimate of the submerged sediment-carrying conduit width, especially around islands and bars, and in shallow areas.
2. The Johannesson-Parker meandering model, discussed in section 3.4.3.3 below, cannot make use of the full width of the river. Specifically, this model causes numerical over- or under-flow in the exponential terms of the equation for u1b at widths over 800 m and/or flows below 800 m^2/s.
3. The behavior of the meandering river is quite different at different widths. In general, a wider river migrates much more slowly and produces fewer, larger, features, while a narrower river is quite active and produces many smaller features. Since this has an effect on the overall coverage of the river, we have elected to make maximum river width one of the parameters which was varied to generate different simulations as part of this study.

Here is a histogram of the distribution of the width (using 50 bins):

<image caption='Distribution of digitized width'>

We make the assumption that the tail of the distribution (everything over 1531 m wide) is spurious, and is due to islands and other artifacts included in the digitization. If these points are neglected, the adjusted histogram and cumulative distribution is:

<image caption='Adjusted distribution of digitized width'>

<table caption='Cumulative width of river'>

The median width (at 50% cumulative distribution) is 693.15 m. We have tried a constant 'effective' width of 400 m in some simulations, which is only as wide as about 18% of the measured values, but this has produced unrealistic behavior. Therefore we have used a minimum maximum width of 500 m in the full set of simulations, while 800 m was the maximum permitted by the JP89 model at all flow values. An additional modification was made to 'soft clip' the river width to a maximum value, rather than simply truncating all widths greater than the cutoff value. This has the effect of maintaining variation in the river width in its widest parts, rather than replacing the widest parts (which comprise a significant fraction of the overall river) by a smooth channel of constant width.

<image caption='Variable adjustment of river width'>

3.4.3 Meandering motion

There appear to be 3 general requirements for river meandering, irrespective of the specific model used:

1. The migration rate depends, as its primary variable, on the local curvature or a derived function of the local curvature, at each point on the river, in the minimal form a * f(c(s))^b for some constants a and b. Meandering models fall into two general classes based on the type of function of curvature used:
   I. So called 'force based' models, which relate migration to physical quantities such as mass, acceleration, and centripetal force.
   II. So called 'speed based' models, which relate migration to the speed of the flow or difference between the speeds of the flows near the two banks of the river.
2. The curvature or derived function must be spatially smoothed (integrated) over some finite interval of the river.
3. The curvature or its derived quantity is usually calculated upstream of its point of application, and therefore has a delayed action on the downstream migration of the river.

The last 2 requirements can be satisfied by having the function f() perform a spatial integration of curvature (e.g. by averaging the upstream curvature along some interval of the river). This is accomplished via the 'ldist' and 'sdist' variables of the effective curvature calculation, which control how far upstream, and over what interval, this integration is performed.

For this study, we have used three different, but related, speed-based meandering methods:

1. Simple curvature.
2. Circumferential speed.
3. Johannesson-Parker (as specified in their 1989 paper).

These three methods are described below.

3.4.3.1 Simple curvature (SC)

In this method, the migration rate of the river is simply proportional to the effective curvature calculated for each point:

```
Rm = a * Ce(s)^b
```

for constants a and b (b is usually set to 1.0, and a is empirically derived by setting the initial erosion rate to a specific value), and for effective curvature Ce at downstream distance s.

Curvature is considered to be 'positive' in sign wherever the river bends to the left, and 'negative' in sign wherever the river bends to the right. At each point on the river, the cross-stream direction is given by the radius vector of the circular arc passing through the point, and has a positive sign toward the left bank, and a negative sign toward the right bank. The migration displacement of a point is in the opposite direction of the sign of its curvature, along the cross-stream vector:

```
dxy = -Rm * left * dt
```

where 'left' is a vector directed toward the left bank as descrbed above, and dt is the simulation step size in years. Simulations based on simple curvature tend to produce small circular loops which may be realistic for smaller streams, but are considered unrealistic for wider, faster flows, as they are not based on any physical parameters of the river, but on geometry alone.

<image caption='Simulation based on curvature alone'>

3.4.3.2 Circumferential speed (CS or sigma)

This simplest speed-based method approximates the channel at each point by a circular arc of non-zero width, and calculates the migration rate as proportional to the excess speed of the flow on the circumference of the outside bank in the bend, as if each radial element of the flow was rigid and traversed the bend at the same angular rate, so that small volumes situated at the larger radius of the outside bank moved faster to cover the longer circumference in the same amount of time as those near the inside bank.

In this case the migration rate is given by:

```
Rm = a * excess_speed
```

where

```
excess_speed = flow / (width * depth) * ((radius + width / 2.0)
               / radius - 1.0)
      radius = 1.0 / Ce(s)
```

for a given flow (in m^3/s), width, depth, and radius of curvature. This excess speed, which is greater than the average speed at the center of the flow, erodes more material from the outside bank, while the below-averge speed at the inside bank causes material to be deposited. Thus, there is an overall migration of the channel toward the outside of the bend. The transfer of material from the outside bank to the inside bank is the basis of channel migration in all the speed-based methods.

This hypothesis is often correct, and generally in accordance with observation, but not for the reasons stated above. Nevertheless, this method produces more realsitic meandering geometry and loop shape than that due to curvature alone, and is also based on the physical parameters of width, depth, and flow, as well as curvature.

<image caption='Similation based on circumferential speed'>

3.4.3.3 Johannesson-Parker 1989 (JP89 or u1b)

The JP89 meandering model is based on several physical input values, including width, depth, flow, bed grain size, and water surface slope, as well as the curvature at each point along the river. This model makes use of 2 coupled second-order differential or integral equations of the stream distance variable s, the latter of which have been implemented by the authors specificaly for this study. In addition, the curvature used at each point is the effective curvature, which is calculated as an additional integral of local curvature along a short interval of the upstream river at each point.

The JP89 method modifies the basic sigma value of the circumferential speed method above to produce a more complicated expression for the speed perturbation u1b at the outside bank of a bend which includes linear terms from the secondary part of the flow which is perpendicular to the stream direction. The JP89 implementation we are using is based on the integral equations 45 and 46 from the Johannesson-Parker 1989 paper:

<image caption='JP89 integral equations'>

In general, these equations form a second order initial value problem. Knowing the values of sigma_s and u_1b at the start of the river (for s = 0):

```
sigma_0 = abs(Ce) * (width / 2.0)
 u_1b_0 = sigma_0
```

allows one to iteratively calculate them for all other values of s downstream. If the ratio of outer bank speed to center speed in a bend is (radius + width/2) / radius, as it is in the circumferential speed method of the previous section, then the excess speed at the outer banks is given as:

```
u1b = ((radius + width / 2) / radius - 1)
    = (width / 2) * (1 / radius)
    = b * C_tilde
    = sigma
```

This is why the circumferential speed method is also called the 'sigma' method, as it is a simpler version of JP89 where the cross-stream contribution to the outside bank speed perturbation is not included. Unfortunately, the two integral equations shown above cannot be used for long rivers since the values of the curvature integrand and integral are large and potentially unbounded, due to the presence of s in the exp() function. Therefore, they quickly overflow or underflow for rivers s values greater than a few km, depending on the other input variables. To alleviate this problems, we have made the following modification to these equations.

Our implementation uses equations 45 and 46 as above, but rather than performing the entire integrals for all values of s from 0 to phi at each step, we calculate only one term of the integration for each sigma_s and u_1b, and then reset sigma_s_0 and u_1b_0 to the previous values from the last phi calculation. This produces the same values for u_1b as the full intergals, but reduces the magnitudes of the integrands and integrals due to the exponential terms, and thus stabilized the calculations for long rivers. It is also a bit faster to perform that the full intergals.

To test these implementations, we simulated a circular channel using the parameters listed in run F2 of Table 2 in the JP89 paper [ref xxx]:

<table id=6>

According to figure 5 of the paper, the observed values of u1b for this channel should be about 0.2 (that is, about 1.2 times the center speed). The circumferential speed method produces the following output:

<table id=7>

yielding a constant u1b of 0.111... (1/9) at all points along the curve. The JP89 implementation (using both the full integral and our modification) produce identical results:

<table id=9>

yielding limiting values of about 0.18. Although this is not exact, it is much closer than the value calculated from sigma alone. Also note that the JP89 calculations does not yield a constant value for u1b immediately, but requires a 'start up' period, and produces the final u1b value asymptotically after a finite delay in s. This appears to be a general characteristic of the JP89 model, and was also seen in the test river simulations shown below. At present, it is not know whether this represents a problem in the JP89 model, as a time or phase delay is often associated with coupled second order systems such as electrical signal filters.

Here is an example of a JP89 river simulation:

<image caption='JP89 simulation'>

High flow and migration rates lead to more activity of the river, but also a kind of 'stylized' geometry:

<image caption='JP89 simulation'>

Here is a more reasonable JP89 test:

<image caption='JP89 simulation'>

Decreasing the flow leads to smaller loops and more erosion:

<image caption='JP89 simulation'>

Increasing the flow increases the loop size, but decreases the erosion rate:

<image caption='JP89 simulation'>

The migration rate can be increased to compensate for this:

<image caption='JP89 simulation'>

As the flow and migration rate are increased further, the geometry becomes stylized:

<image caption='JP89 simulation'>

Two other potential problems with the JP89 model appear to be:

1. The values of u1b go from far less than 1 to many times 1, whereas they are expected to be near to and just greater than 1 for realistic bends and curvatres. Nevertheless, this does not appear to adversely affect the meandering behavior of the simulation.

<table id=5>

2. There is an inherent numerical instability to the JP89 model that causes regions of low curvature, or where the curvature changes sign, to degenerate into oscillation or unbounded random motion. This instability appears to be in addition to the innate instability of straight flowing segments which leads to normal meandering behavior.

<image caption='Oscillation of JP89 output'>

<image caption='Transition to random behavior in JP89 output'>

Again, resonance at particular frequencies is often a property of coupled second order systems such as bankd-pass filters. In this case, the JP89 behavior might be more akin to a system of masses connected by springs which absorb energy at a specific set of frequencies. In the JP89 simulations used in this study, we have taken several numerical steps to attempt to damp this behavior, such as smoothing and reinterpolating the output of the JP89 meandering module. However, some resonances continue to appear, generally for slow flows in wider rivers. This subject will be mentioned again in section 4.6.1 below.

3.4.4 Limitations to meandering methods

The single-thread meandering model used in this study is clearly not adequate to accurately represent a river such as the Missouri, primarily because:

1. It reduces the geometry of the river to a single idealized channel defined by a centerline and width and having a trapezoidal cross-section, rather than the often braided or multiple-channel watercourse which contains numerous islands, chutes, and submerged and surface bars.
2. It does not track the transport of sediment within the flow from where it is eroded from the banks or bottom to where it is eventually deposited downstream. The JP89 method in particular makes the simplistic assumption that material eroded from one bank of the channel is immediately carried to and deposited on the opposite bank, so that the rate of migration is the same for both banks, and the width of the channel does not change.

The JP89 method only accepts physical inputs in a fairly narrow range of values, otherwise the values of the exponents of s computed in the integral equations either underflow of overflow. In general, the JP89 method 'likes' very narrow streams with fairly high flows. To use the JP89 method with the MNRR over a simulated period of 100 years, we have, for example, had to limit the maximum width to 800 m, the minimum flow to 800 M^3/s, and the minimum depth to 2.75 m. In addition, we have been unable to find a set of initial conditions which produce migration behavior that is in general similar to the MNRR, as seen in the maps and photos. Specifically, the JP89 method does not produce loops and other features that closely resemble those seen in the MNRR without some manual supervision as to the values of certain input parameters.

We have, for example, attempted to create JP89 simulations specificaly having larger 'loops' similar to those seen in the 1890 maps of the MNRR, smoe of which currently exist as oxbow features in the 1957-19956 maps and 2006 satellite images. This does not present any great difficulty, as it is only a matter of picking the proper distances used for calculating effective curvature, the median distance being approximately one river width. However, one downside to generating bigger loops seems to be that the river may then 'hug' the valley wall for long periods and distances, as there is then less variation in curvature to make it change direction and migrate away from the wall. Thus, loop size appears to be in instance where there is a tradeoff between similarity to the actual river, and potential stagnation of the migration of the stratghter segments which are near to the boundary.

In general, we have found that artificially 'clipping' the river width to a maximum value of 600 m or below creates simulations which are more active and more closely resemble the shape of the MNRR. This observation corroborates our hypothesis that a narrower channel is more representative of the actual meandering width, especially in segments where we have disregarded islands, shallows, or multiple channels in the digitization. We feel that the water surface and bank width are a deceptive kind of 'event horizon' which hide from view the fast-flowing, narrow, submerged conduits which are actually responsible for sediment exchange and the migration of the overall river. One potential consequence of this hypothesis is that there might be a second meandering planform beneath the visible planform, and that the surface width of the river may be just the 'hull' or 'envelope' of the range of the active submerged planform. We examine this possibility further in the multiple-thread simulations of section 5.4.3.1.

3.4.5 Erosion rate

The erosion rate, in hectares per year per kilometer of river length, is the primary independent variable governing the migration rate of the simulations in this study. In the equations for migration shown in section 3.4.3 above, the exponent b is set to a value of 1.0, and the constant factor a is determined empirically by setting the initial erosion rate of the river to a specific value, which has been determined by observation of the actual river, and is nominally estimated at 1.2 ha/yr/km.

In all simulations, the erosion rate is determined after each dt step in a dynamic way that involves the motion of every point on the river, rather than statically as the difference between final land area minus initial land area. In particular, the surface area eroded between any two points on the river during a particular time interval dt is estimated as the area swept out by a quadrilateral whose vertices consist of the locations of points i and i + 1 at times t and t + dt.

<image caption='Eroded area between 2 points during 1 time step'>

The total area eroded by the river during a single time step is the sum of the individual areas calculated between every two consecutive points. The erosion rate at that time in the simulation is therefore this total are divided by the length of the river, multiplied by the time step. The degenerate quadrilateral case, where points i and i + 1 move in opposite directions, is not considered, since in most cases consecutive points of the river will move in the same direction. Thus, this estimate is an upper bound of the actual rate of erosion.

<image caption='Degenerate case not considered'>

Thus, whenever points of the river are in motion, they are considered to be eroding surface area from the surrounding plain. The JP89 meandering model makes the assumption that all material eroded from one bank of the river will be immediately deposited on the other bank, but this is not a realistic assumption. Although some of this material may be deposited on the opposite, or the same, bank, it is most likely to do so carried somewhere downstream by the flow, possibly out of the simulation domain. The present study makes no attempt to estimate the fraction of sediment redeposited within, or carried beyond, the simulation domain, and therefore consideres all material removed by the migration of the river to be 'eroded', again making this estimate an upper bound.

Therefore, even if the average migration of a finite segment of the channel over time is negligible, and the measured surface area at the strand does not appear to change, the discrete motion of individual points within that segment will continue to erode material. At present, it is assumed that there is an inexhaustible supply of erodable material, and that all erosion is limited to the surface and due only to the lateral migration of the channel. These are unrealistic assumptions, but they do not affect the outcome of this study. In more realistic simulations, computational bookkeeping to track and balance the transport of mass and volume of eroded and potentially redeposited material would be required.

The initial erosion rate river can be automatically set to a given value by the simulation program by varying the migration rate using a binary search. This is done at the start of every simulation in the large populations of runs used in this study. Here is typical output of setting the initial rate to 1.2 ha/yr/km:

<table id=20>

In this case, the migration rate was changed from 1.0 to a final value of xxx after 25 steps. This is very quick to accomplish, as only the meandering and erosion calculations need be performed each step, without boundary or cutoff checks, or reinterpolation. Once initialized, the migration rate factor is held constant for the duration of the simulation. Although the initial erosion rate is set to a known value, this value may increase or decrease at any time during the course of the simulation, and no additional attempt is made to hold it at a constant value.

3.4.6 Area coverage & residence time

Every simulation accumulates a 2-dimensional array showing how long the river stays in any one place as it moves across the valley during 100 years. This array is accumulated at a resolution of 0.5 x 0.5 km squares for all areas within the valley boundary, and is saved initialy and also every 10 years during the course of the simulation. At the end of every step of the simulation, all grid squares which contain points of the river (including both centerline and left and right banks as determined by the width of the river), are incremented by the value of dt. Since the river is discretized at a resolution of 100 m, and the maximum half-width of the river used in the simulations is 400 m, a 2-d array with resolution of 500 m is sufficient to capture all points on the river.

At the end of the simulation, this array shows the total amount of time during 100 years that the simulated river occupied every 500 m x 500 m area in the valley, although this is not necessarily continguous time. Incremental times occupied during 10 year intervals can be calculated by subtracting one array from another. All values of this array which are non-zero show the maximum extent of the range of migration during this time. Cutoffs (excised intersecting loops) are included in this array, as well as the active migrating channel. Note that this array may conatin lacunae --that is, areas of zero occupancy contained within an area of non-zero occupancy, indicating that the river has at some time curved back upon itself insufficiently to acquire all intermediate area.

It is expected that the accumulation of such arrays for several thousand simulations will enable the calculation of aggregate time and location statistics that indicate with some confidence the potential migration range of the single actual river and which can be used to generate probability curves (contours) at different fractional values which will define the overall 'migration corridor' of the river as a function of time.

3.4.7 Cumulative eroded area

The dynamic erosion calculation performed in section 3.4.5 above is also accumulated in a 2-dimensional array at 10 year intervals for every simulation, indicating how much surface area and sediment each river erodes from the valley, and where. Arrays accumulated for every simulation will also be combined to estimate the average cumulative erosion of the single river as a function of time and location.

3.4.8 Generation of multiple simulation parameter combinations

The key to the success of the strategy used in this study is the ability to generate an intelligently chosen set of different simulations. The ability to generate and run many different simulations from a single program is provided by the ability to read and parse command files as discussed in section 3.3.4 above. However, the choice of the input variables used, and the ranges and discrete values of those variables, was made manually by running and reviewing the results of many potiential combinations of those variables before a final selection was chosen.

Because we were able to run 100 year river simulations in a reasonable amount of time (about 15-20 minutes each), we were able to explore some of the paramater combinations for the Johannesson-Parker and other speed-based meandering methods (e.g. circumferential speed) in preparation for the large set of runs. As with all curvature-based methods, an enormous variation in river geometry can be produced, including considerable variation in loop size and shape. This is based primarily on different combinations of the variables 'ldist' and 'sdist' which control the integration resulting in effective curvature. We have chosen one of these variations (ldist = sdist = 1.0 river widths) to use as the starting point for the large population of simulations we performed for this study.

<image caption='Comparison of loop size and shape based on ldist and sdist'>

We decided to use both the JP89 and one other meandering method to compare and contrast aggregate results. The circumferential speed method is essentially a simpler version of JP89 based on the same idea, without making use of the perturbation in far bank speed due to secondary cross-stream flow. All aggregate simulations were subjected to the constraint that the initial erosion rate be close to 1.2 ha/yr/km, as reported in [ref xxx]. Most of the 'interesting' river simulations have erosion rates far in excess of this value (up to 20-30 ha/yr/km). Simulations limited to realistic erosion rates are considerably less dramatic, with far fewer cutoffs, but still consume planform area.

The response of the JP89 model to increased flow is somewhat counter-intuitive. Although the loop size does increase, the erosion rate decreases with increased flow. The migration rate must be turned up in order to create an initial erosion rate of 1.2 ha/yr/km, but that rate generally decreases over the simulation time (whereas with other meandering methods the erosion rate usually increases with time as the river increases in length and sinuosity). Both decreasing the depth of the river or increasing the bed grain size appear to have the effect of making the river more 'active', although decreasing the depth also magnifies the differences in activity seen between the smaller and larger grain sizes.

<image caption='Comparison of JP89 response to flow'>

<image caption='Comparison of JP89 response to depth'>

<image caption='Comparison of JP89 response to grain size'>

more...